Every Student Counts

Professional Development Guide
Elementary School Level

Algebra

Year 2 - Day 1

Iowa Department of Education

Elementary Session – Facilitator's Plan
Year 2 Day 1

Content Goals:
NCTM- Algebra Standard

Represent and analyze mathematical situations and structures using algebraic symbols

- (K-2) Illustrate general principles and properties of operations, such as commutativity, using specific numbers;
- (K-2) Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.
- (3-5) Identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers;
- (3-5) Represent the idea of a variable as an unknown quantity using a letter or a symbol;
- (3-5) Express mathematical relationships using equations.

Use mathematical models to represent and understand quantitative relationships

- (K-2) Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.
- (3-5) model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

Analyze change in various contexts

- (K-2) Describe qualitative change, such as a student's growing taller;
- (K-2) Describe quantitative change, such as a student's growing two inches in one year.
- (3-5) Investigate how a change in one variable relates to a change in a second variable;
- (3-5) Identify and describe situations with constant or varying rates of change and compare them.

Principle Focus: Equity

Process Goal:
Connections Standard K-12

- Recognize and use connections among mathematical ideas;

- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;

- Recognize and apply mathematics in contexts outside of mathematics.

Overall Teaching Goal:
Teaching and learning mathematics through problem solving

Activity	Description for facilitator	Time (min)	Teacher Masters (TM-) & Materials
1. Welcome	Discussion of agenda, goals and sessions for the day	5	**Summary of the Session** **TM-1A** Year Two Outline (2005-06)
2. Sorting Tasks By Cognitive Demands	• Refining understanding of mathematical tasks for problem-based instruction	60	**TM- 1B** NCTM Teaching Standard 1 **TM- 2** Levels of Cognitive Demand **TM- 3** Problem-based instructional task definition Task pages and answer matrix from *Perspectives on the Teaching of Mathematics, pg. 51-59*
3. Algebra Content – Using algebraic symbols	• Equality Exercise (K-2)	20	**TM- 5** Card Set
4. Algebra-Problem-based Instructional Task 1 (PBIT)	• Acrobats, Grandmas and Ivan (3-5)	30	**TM- 6** PBIT for Acrobats, Grandmas and Ivan "A Mathematical Tug of War" from *Math for Smarty Pants* Collections from *Algebra Thinking* **TM-7** Number Tricks
5. Meaningful Distributive Practice (MDP) exercises	• K-2 Equality • 3-5 Algebraic symbols	30	**TM-8** MDP **TM- 9** MDP The Grapes of Math, Page 82 from *Navigating Through Algebra 3-5*
	• LUNCH	45	
6. Algebra-Problem-based Instructional Task 2	• And We All Go Marching	60	*Navigating Through Problem Solving and Reasoning Grade 3 pg. 12-20.* Linking cubes Centimeter grid paper
7. Algebra resources	Developing understanding of the big ideas of Algebra using *Groundworks*	30	Selected pages from *Groundworks – Grades 1-5* and *Algebra Thinking First Experiences*
8. Problem-based Instructional Task 3	• *Two of Everything*	30	**TM-10** Definition of MDP **TM-11** PBIT Task from *Two of Everything* Book **TM-12** MDP for functions Calculator Centimeter cubes Graph paper
9. Processing readings	• Reading summary and discussion	30	**TM- 4** Reflection Questions from Reading Assignment for Day 1
10. Reflection/ homework assn	• Evaluation and reading assignments	15	**TM-13** Reading Assn. for Day 2 **TM-14** Practice Assn. for Day 2

Facilitator's Tool for Planning the Session

Equipment and materials the facilitator should bring:

Equality Card Set
Perspectives on the Teaching of Mathematics
Math for Smarty Pants
Algebra Thinking First Experiences, Creative Publications
Navigating Through Algebra 3-5
Navigating Through Problem Solving and Reasoning Grade 3
Groundworks –Grades 1-5
Two of Everything Book
Graphing paper, pencils, rulers and overhead graphing blanks to allow participants to share work.

Equipment and materials participants should bring:

Tub of linking cubes, centimeter cubes, *Navigating Through Reasoning Grade 3*; four function calculators.

Prepare as handouts:

TM-1A Elementary Year Two Outline (2005-06)
TM- 1B NCTM Teaching Standard 1
TM- 2 Levels of Cognitive Demand
TM- 3 Problem-based Instructional Task Definition
TM- 5 Equality Card Set
TM- 6 Problem-based Task for Acrobats, Grandma and Ivan
TM-7 Number Tricks
TM-8 Meaningful Distributed Practice for Algebra K-2
TM-9 Meaningful Distributed Practice for Algebra 3-5
TM-10 Definition of MDP
TM- 11 Problem-based Instructional Task from *Two of Everything*
TM-12 Meaningful Distributed Practice for functions
TM-13 Reading Assignment for Day 2
TM-14 Practice Assignment for Day 2
TM-14 Meaningful Distributed Practice template
Obtain and bring along this article from the NCTM website: Karen P. Falkner, Linda Levi and Thomas P. Carpenter. *Children's Understanding of Equality: A Foundation for Algebra.* NCTM: December 1999, Volume 6, Issue 4, Page 232.
http://my.nctm.org/eresources/article_summary.asp?from=B&uri=TCM1999-12-232a

Prepare as overheads

TM- 3 Problem-based Instructional Task Definition
TM-4 Reflection Questions from Day 1 Reading Assignment
TM-7 Number Tricks
TM-8 Meaningful Distributed Practice for Algebra K-2
TM-9 Meaningful Distributed Practice for Algebra 3-5
TM-12 Meaningful Distributed Practice for functions
"Collections" problem pg. 71 from Algebra First Experiences

Activity 1: Welcome

Time: 5 minutes

Overview and Rationale:
Conducting the Activity:

Summary of Session— Day Two of Algebra Strand

- In the first activity, participants will refine understanding of problem-based instructional tasks through a sorting activity.
- In the next activity, participants will develop expertise in teaching equality concepts and properties of mathematics related to generalized rules of algebra.
- Meaningful Distributed Practice will then be featured and participants will practice developing MDP for algebra.
- The NCTM Algebra Standard will be illustrated with personal and commercial resources including *Navigating through Algebra*, *Algebra Thinking First Experiences*, and *Algebra*
- *Puzzles and Problems* with group activities and problem-based learning.

Materials:

TM-1A Year Two Outline (2005-06)

TM-1A

Elementary Year Two Outline 2005-06

	Day 1 September 13-14, 2005	**Day 2** November 1-2, 2005	**Day 3** January 24-25, 2006	**Day 4** March 28-29, 2006
NCTM Content Standard 1	Algebra Representing using math symbols; Modeling; Analyzing change	Geometry Analyzing 2D and 3D shapes and characteristics	Geometry Coordinate geometry	Geometry Transformations and visualizing
NCTM Content Standard 2		Measurement Understand measurable attributes of objects and the units, systems, and processes of measurement	Measurement Apply appropriate techniques, tools, and formulas to determine measurements	
Mathematical Activities	K-2 Algebra • Illustrate general principles and properties of operations, such as commutativity, using specific numbers; • Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations. • Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols. • Describe qualitative change, such as a student's growing taller; • Describe quantitative change, such as a student's growing two inches in one year	Measurement K-2 • Recognize the attributes of length, volume, weight, area, and time; • Compare and order objects according to these attributes; • Understand how to measure using nonstandard and standard units; • Select an appropriate unit and tool for the attribute being measured. Geometry K-2 • Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes; • Describe attributes and parts of two- and three-dimensional shapes; • Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.	Measurement 3-5 • Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes; • Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles; • Select and use benchmarks to estimate measurements; Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms; Develop strategies to determine the surface areas and volumes of rectangular solids Geometry K-2 • Describe, name, and interpret relative positions in space and apply ideas about relative position; • Describe, name, and interpret direction and distance in navigating space and apply ideas	Geometry K-2 • Recognize and apply slides, flips, and turns; • Recognize and create shapes that have symmetry. • Create mental images of geometric shapes using spatial memory and spatial visualization; • Recognize and represent shapes from different perspectives; • Relate ideas in geometry to ideas in number and measurement; • Recognize geometric shapes and structures in the environment and specify their location. Geometry 3-5 • Predict & describe results of sliding, flipping, and turning two-dimensional shapes;

	3-5 • Identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers; • Represent the idea of a variable as an unknown quantity using a letter or a symbol; • Express mathematical relationships using equations. • Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions. • Investigate how a change in one variable relates to a change in a second variable; • Identify and describe situations with constant or varying rates of change and compare them.	Geometry 3-5 • Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes; • Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids; • Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes; • Explore congruence and similarity; • Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.	about direction and distance; • Find and name locations with simple relationships such as "near to" and in coordinate systems such as maps. Geometry 3-5 • Describe location and movement using common language and geometric vocabulary; • Make and use coordinate systems to specify locations and to describe paths; • Find the distance between points along horizontal and vertical lines of a coordinate system.	• Describe a motion or a series of motions that will show that two shapes are congruent; • Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs. Build and draw geometric objects; • Create and describe mental images of objects, patterns, and paths; • Identify and build a three-dimensional object from two-dimensional representations of that object; • Identify and draw a two-dimensional representation of a three-dimensional object; • Use geometric models to solve problems in other areas of mathematics, such as number and measurement; • Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.
NCTM Principle	Equity	Technology	Teaching	Learning
NCTM Process Standard	Connections Reasoning and Proof	Reasoning and Proof Representation	All	Communication Representation
Assessment	Formative use of Meaningful Distributed practice	Interviews for conservation of measurement concepts	Questioning strategies for Problem-based Instruction	Journaling Analyzing student work
Technology /Manipulative Tools		Java Applets on the Internet; nets	Java Applet on transformations Geoboard explorations	Link cubes and isometric drawings

Every Student Counts means . . .

Teach for Understanding
and
Focus on Meaning

Problem-Based Instructional Tasks &
Teaching through Problem Solving

Meaningful Distributed Practice of Concepts, Skills, and Problem Solving

Today's Goals . . .

Content Goal: *Algebra*
Process Goal: ***Connections & Context***

Today's Objectives . . .

- ***xxxxxxxxxxxxxx***
- ***xxxxxxxxxxxxxxxxxx***

Activity 2: Sorting Tasks by Cognitive Demand

Time: 60 minutes

Overview and Rationale:
Participants sort and classify tasks for cognitive challenge level. Discussion afterwards helps teachers clarify what constitutes a problem-based task. Teachers will be able to connect this experience to teaching for understanding using representation.

Conducting the Activity:

A wide range of activities, problems and resources exist for teachers to use and to choose from in our work with children. How do we choose?

The Professional Teaching Standards for teaching mathematics have given us some guidance. Using our definitions for a problem-based task and the standards given, what similarities do you see?

Through this activity we hope a clearer and more refined understanding of problem-based instructional tasks will develop in order to help us understand some of the key aspects of a task we should attend to. A powerful tool for learning is classifying. We are going to sort a number of tasks according to cognitive demand. Let's look at what Smith and Stein, 1998, have developed as guidelines.

1. Share criteria from Smith and Stein using overheads TM-2 and examples for teach category using overhead TM-3. Instruct teams to create a poster divided into 4 quadrants and labeled with each category.
2. Distribute post it notes and task pages to each table. Instruct participants read each task in turn and suggest a category for it. If everyone agrees, label a post it with the letter of the task and put it on the poster inside the quadrant for the corresponding category.
3. Teams should post sheet on wall when finished. Ask teams to discuss low level tasks we have in agreement first. Discuss reasoning used. . Continue with high level tasks next. Areas of disagreement help to refine criteria. Refer to page 45-47 of *Perspectives on the Teaching Mathematics Professional Development Companion* for summary of task activity. Answer matrix should be given after discussion.
4. Pose questions for discussion afterwards: Is there a difference between "level of cognitive demand" and "difficulty"? What effect does context, prior experience have on level of cognitive demand?
5. Pause for reflection and have teachers write new learnings on feedback sheets from this activity.

Materials:
TM-1B NCTM Teaching Standard 1
TM-2 Levels of Cognitive Demand
TM-3 Problem-based instructional task definition
Task pages and answer matrix from *Perspectives on the Teaching of Mathematics, pg. 51-59*

TM-1B

NCTM Teaching Standard 1

The teacher should pose tasks based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics

and that

- engage students' intellect;
- develop students' mathematical understanding and skills;
- stimulate students to make connections and develop a framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to and draw on students' diverse background and dispositions;
- promote the development of all students' disposition to do mathematics.

TM-2

Levels of Cognitive Demand

Lower-level demands (memorization)

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, ruled, formulas, or definitions being learned or reproduced.

Lower-level demands (procedures without connections)

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanation or explanations that focus solely on describing the procedure that was used.

Continued on next page

Higher-level demands (procedures with connections)

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problems situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics)

- Require complex and non-algorithmic thinking - a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

TM-3

Definition: Problem-Based Instructional Tasks

- Help students develop a deep understanding of important mathematics
- Are accessible yet challenging to all students
- Encourage student engagement and communication
- Can be solved in several ways
- Encourage the use of connected multiple representations
- Encourage appropriate use of intellectual, physical and technological tools

Activity 3: Algebra Content – Using Algebraic Symbols
Equality Exercise (K-2)

Time: 20 minutes

Overview and Rationale:

Jenny Johnson is supposed to send me her write up and has not done so.

Conducting the Activity:

Materials:
TM-5 Equality Exercise Card Set

TM-5

Equality Card Set

2+1	4+1	=
1+0	3+3	=
7+3	2+4	=
5+5	4+5	=
3+1	2+5	=
1+1	2+7	=
0+1	2+3	=
4+6	3+6	=
2+2	5+3	=
1+2	1+7	=
2+0	4+3	=
4+4	3+5	=

T-M5 Equality Exercise Card Set continued

12-4	9-4	11-5
16-9	10-2	13-7
13-6	8-3	17-8
14-5	9-5	18-8
6-5	8-6	5-3
15-5	8-4	9-6
4-3	6-3	7-4
5-2	4-2	5-4
10-5	10-6	10-7
12-2	16-6	13-4
12-3	14-6	9-3
7-6	7-1	10-1

T-M5 Equality Exercise Card Set continued

2x2	2x1	10x1
4x1	5x2	3x1
1x1	3x3	2x3
4x2	1x9	1x6
7x1	5x1	8x1
4÷2	8÷4	18÷3
12÷3	6÷6	70÷10
9÷3	3÷3	18÷2
30÷3	16÷2	90÷10
15÷3	14÷2	12÷2
40÷5	10÷2	5÷1
÷	÷	÷

Activity 4 – Problem-based Task (PBT) 1, "Acrobats, Grandmas, and Ivan"

Time: 30 min.

Overview and Rationale:
This is a problem-based instructional task for algebra that highlights the use of a variable and manipulatives to make complex relationships easier to represent. This activity lays the foundation for exploring concepts for systems of equations.

Conducting the Activity:

1. Read the problem aloud and check for understanding of the relationships. Ask participants to solve it using any strategy they can and record on chart paper their answers and solution method.
2. Allow 10 minutes to work in small groups.
3. Present selected strategies to the group. Notice how many people used representations for Ivan, acrobats and grandma. Relate this to the use of a variable to represent mathematical relationships.
4. Using manipulatives to show relationships are part of the resources provided last year. Show Collection problem page 71 from *Algebra Thinking First Experiences* and ask participants to solve with cubes.
5. Show how the various collection problems on pages 59-71 become progressively more challenging, beginning with easy numbers, then fractional representations, and finally students find a systematic way of determining the number of insects. Can you write an equation or two for the relationships? How are these like the Ivan problem? In *Navigating Through Algebra 3-5* see "Catch of the Day" page 41.
6. Try a **number trick** using an envelope, beans and cubes. Have one participant put a secret number of beans in an envelope. Follow the steps on the overhead and see why the answer is always three. Use a variable to represent each step.
7. Relate the idea of variables and equations as a way to model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions about a situation.

Materials:
Math for Smarty Pants (lesson adapted from ***Math for Smarty Pants by Marilyn Burns)***
TM–6 Problem-based Task for Acrobats, Grandma, and Ivan from A Mathematical Tug of War in Marilyn Burns' *Math for Smarty Pants* SANDI CHECK THIS.
TM-7 Number Tricks

Problem-Based Instructional Task Lesson Plan

Lesson Topic: Using Algebraic Symbols
Grade Level/Course: Grade 4-5
Objective: Students will generate strategies for recording that involve symbolic notation to help solve this problem.
Pre-requisite Knowledge: None.
NCTM Standard(s): (shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

Materials Needed:

Audio-visual:
Manipulatives:
Literature: ***Math for Smarty Pants***
Technology/Software:
Other:
Lesson adapted from ***Math for Smarty Pants by Marilyn Burns***

TM-6

Acrobats, Grandma, & Ivan
Problem-Based Instructional Task Lesson Plan

Main Lesson Development:

Lesson Topic: Using Algebraic Symbols **Grade Level/Course:** Grade 4-5
Objective: Students will generate strategies for recording that involve symbolic notation to help solve this problem.
Pre-requisite Knowledge: None.
NCTM Standard(s): (shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

What do you know in this situation? What can you figure out from this?
Would acting it out help?
What would happen if you had only part of the teams playing tug of war?
Share: When ready, share progress with whole group and analyze strategies. Note those who used symbolic notation to record and discuss whether that was a helpful or efficient strategy. Identify this useful use of a variable.
Summarize and Clarify: Make the following points with the class: Algebraic notation can be very useful for understanding complex problems. When we have equal relationships, they can be substituted in new situations to help us solve them.

Checking for Understanding (Formative Assessment):

- **What will you assess?** Understanding of the use of variables to solve problems

Assessment Problem Ask students to find who would be the winner in the following case: Ivan and 2 acrobats were tied with 8 grandmas in the first round. Then in the second round 2 acrobats tied with 3 grandmas. What would happen in the third round with 5 grandmas against Ivan?

- **How will you assess it?** Students will complete another problem similar to the first.

Reflection After Teaching the Lesson:

- How did the students perform?
- How will you use this information to guide future instructional decisions?

Modifications: Some students may visualize equality using a balance scale and simple exchanges for shapes or weight first before trying this story.

Extensions: Challenge the students to write more stories from this situation. Can the students write an equation in which the Grandmas would win without Ivan?

TM-7

Activity 5 – Meaningful Distributive Practice (MDP) Exercises

Time: 30 min

Overview and Rationale:
Participants will illustrate general principles and properties of operations. They will use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations and use them in distributive practice exercises. They will identify such properties as commutativity, associativity, and distributivity and use them to mentally compute with whole numbers and fractions. They will represent the idea of a variable as an unknown quantity using a letter or symbol and express mathematical relationships using equations.

Conducting the Activity

1. Distribute the distributive practice template and review the use of the distributive practice procedure. Identify concepts in algebra that can be used in distributive practice e.g. commutativity, associativity, distributivity and mental math exercises.
2. A piece of literature, *The Grapes of Math*, can be used to show how the properties and number combinations can make counting objects an easier task. Read several pages from the book and show pictures. Ask participants to describe how they counted the objects.
3. Hand out page 82 from *Navigating* and let participants find patterns and number sentences. Are they equivalent? Do any show the relationships between addition and multiplication? The Associative property? Etc.
4. Show a MDP page designed around these experiences.

Materials
Pages 48-50, 82 in Navigating through Algebra in Grades 3-5
The Grapes of Math
TM-8 Distributive Practice template

TM-8

Meaningful Distributed Practice

Distributed Practice and Questions: **Grade Level ___1___**

Big Idea(s)

Algebraic thinking focusing on equality and the = symbol

MDP Activity 1	MDP Activity 2	MDP Activity 3	MDP Activity 4	MDP Activity 5
5+5=10	5+5=10 6+4=10 3+7=10 3+2+5=10 10=2+8 10=11-1 100-90=10 15-5=10 1+1+1+1+1+1+1+1+1+1=10 4+6=10 5+1+1+1+1+1=10	5+5=10 6+4=10 3+7=10 3+2+5=10 10=2+8 10=11-1 100-90=10 15-5=10 1+1+1+1+1+1+1+1+1+1=10 4+6=10 5+1+1+1+1+1=10	5+5=10 6+4=10 3+7=10 3+2+5=10 10=2+8 10=11-1 100-90=10 15-5=10 1+1+1+1+1+1+1+1+1+1=10 4+6=10 5+1+1+1+1+1=10	6+4=4+6 true 3+7=5+5 true 11-1+3+7 true 2+3+5=15-5 true 2+8=3+8 not true
Questions: What does this number sentence say? Can you show me how this looks with cubes? Can you tell me other ways we can make 10? Any ways to make 10 other than using addition?	**Questions:** Rewrite some of the equations and ask if the are true or not true; i.e., 15-5=10 to 10=15-5 4+6=10 to 4+7=10 What does the = mean?	**Questions:** Rewrite wome of the equations and ask if true or not true; i.e., 6+4=4+6 3+7=5+5 11-1=3+7 2+3+5=15-5 2+8=3+8	**Questions:** Using post-it notes, cover a numeral in each of the number sentences on the chart. Ask what number belongs and why.	**Questions:** Using post-it notes, cover a numeral in each number sentence and ask the children what belongs and why. Extension: Write naked number sentences that children have never seen and discuss responses.

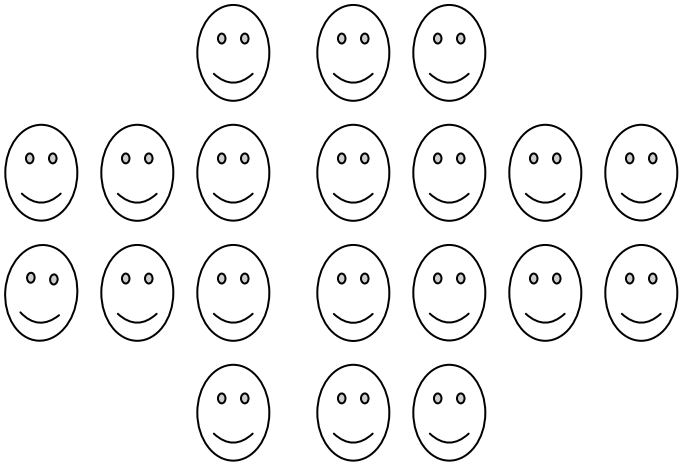
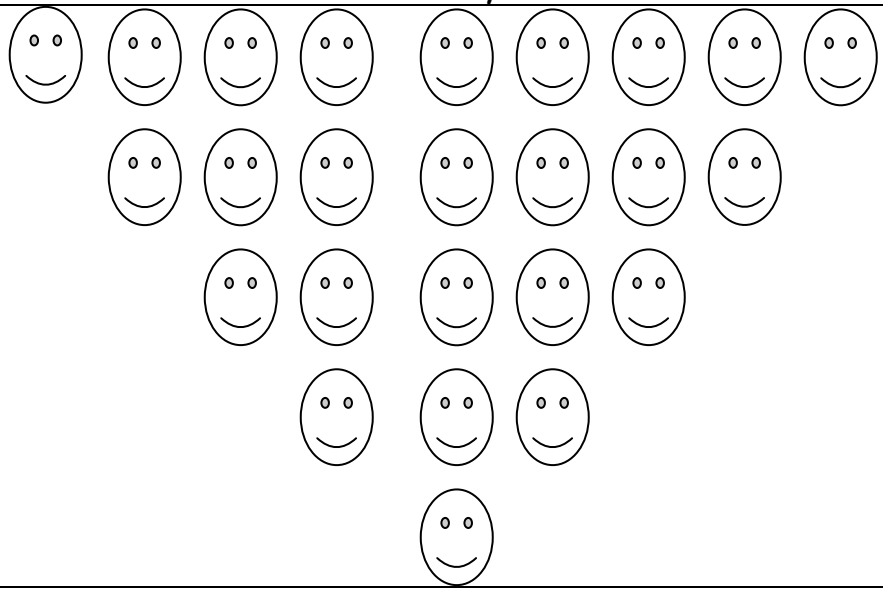
TM-9

Meaningful Distributed Practice

Distributed Practice and Questions: Grade Level ____

Big Idea(s)

Equivalent numerical expressions represent the commutative and associative properties of operations. We can use symbols to express the general rule for these properties.

MDP Activity 1	MDP Activity 2
	
Questions: What numerical expressions does this partition give you? 1. (4x3) + (4x2) or 4x5. Write the expression: (a x b) + (axc) = a x (b+c) distributive.2. (2x7) +6 or 6+(2x7). Write the expression: a + (b x c) = (b x c) + a. Is this true? (Commutative Property)	Questions: What are two numerical patterns we can write for this partition? (10+10+5) or (10+5+10) Associative Property or (2x10)+5 Definition of multiplication. (a+a) = (2 x a)

Activity 6: Problem-based Task 2, "And We All Go Marching" Analyze Change in Various Contexts

Time: 60 minutes

Overview and Rationale:
Participants will describe qualitative and quantitative change, investigate how a change in one variable relates to a change in a second variable and identify and describe situations with constant or varying rates of change and compare them.

Refer participants to "Squares Cubed" in *Navigating through Algebra in Grades 3-5* page 64 for a similar activity for grades 3-5.

Conducting the Activity:
Follow the instructions found in the Navigating materials with the participants.

Materials:
Navigating Through Problem Solving and Reasoning, Grade 3, pg. 12-20.
Linking cubes
Centimeter grid paper

Activity 7 – Algebra Resources

Time: 30 minutes

Overview and Rationale:

Participants will become familiar with provided resources to enhance algebra instruction. These activities include solving hands-on systems of equations and addition properties using positive and negative numbers. Using representations with cubes allows children to explore the answers to these puzzles. Each table of participants will explore a sequence of activities for a type of activity and share with their group observations on how this material can be used.

Conducting the Activity

Say, “We have been looking at patterns using manipulatives, but also tables and words. Another good commercial resource for algebra and in this case, patterns, is the Groundworks series by Carole Greenes and Carol Findell. These resources are grade level specific - from first grade to seventh grade. The author says, "Students who enter the study of algebra from an arithmetic-driven program often find the new content confusing and daunting. The main reason for this difficulty with algebra is a lack of preparation. Although the NCTM- has recommended that students begin preparation for the big ideas of algebra during their elementary school years, current mathematics programs do not provide sufficient experiences. *Groundworks: Algebraic Thinking* introduces students to the six big ideas of algebra (representation, proportional reasoning, balance, variable, function, and inductive/deductive reasoning) using interesting, challenging problems."

1. Distribute packets made up of pages from the *Groundworks* series and have tables complete a sequence of activities and share observations.
2. *Navigating Through Algebra* has many of the same kinds of exercises for children. (The masters in the book are misaligned. Use the CD, which should be correct.)
3. Each table processes own topic and shares these key points:
    a. How can this be used in the classroom?
    b. As you reflect on problem-based instructional tasks, how do these materials fit in?
    c. How can we make these more student- centered?

Materials:

Groundworks series, Grades 1-5
Algebra Thinking First Experiences

Activity 8: Problem-based Task 3 "Two of Everything"

Time: 30 minutes

Overview and Rationale:
Participants will explore how a function machine can be introduced to children and used as meaningful distributed practice. Function machines can be used to practice computation, mental math, and properties of numbers.

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Conducting the Activity:

1. Read the book aloud using good questioning to enhance understanding of the function of the magic pot and model for participants the problem-based task associated with this book.
2. Introduce the MDP possibilities for reinforcing properties of math such as Addition of Zero, Multiplication by One, etc. using a function machine.
3. Have participants share variations of the function machine they have experience with.

Materials:
TM-10 Definition of MDP
TM-11 Problem-based Instructional Task from Book *Two of Everything*
TM-12 Meaningful Distributed Practice for functions
Calculator
Centimeter cubes
Graph paper

TM-10

Definition of Meaningful Distributed Practice (MDP)

- Targets an identified need based on multiple data sources
- Helps students develop a deep understanding of a BIG IDEA
- Helps students develop flexibility and fluency with skills and concepts
- Builds on and extends understanding
- Uses problems and activities that help students learn to use multiple representations, and learn to use multiple reasoning strategies
- Uses problems from a variety of contexts so students learn to make connections

TM 11: Problem-Based Instructional Task Lesson Plan

Objective/Benchmark: Students will learn to reason algebraically using function machines.
Title: *Two of Everything* **Grade Level/Course:** Grade 2-4
Pre-Requisite Knowledge: Using paired numbers to find locations on a graph
Previous experience with patterns that grow may be helpful

NCTM Standard(S): (Shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

Materials Needed:
Literature: *Two of Everything* by Lily Toy Hong
Lesson adapted from Lessons for Algebraic Thinking by Marilyn Burns

Main Lesson Development:

Launch: Read *Two of Everything* aloud. Discuss the action of the doubling pot and create a t-chart for doubling and ask students to describe the change in numbers. Describe the patterns using words. Introduce the use of □ and △ to show variables in place of the words. Show

□ = 2 x △
□ = △ + △
□ = △ **x** 2

Invent several rules with one or two steps and share the first in and out pair. Ask students to guess the rule by predicting the next value for the output 4 times. Discuss how to write the rule as an equation using boxes and triangles and words. When students understand the use of t-chart and variables, move to the explore phase.

Explore:
1. Ask selected students to create a rule for the class to try and figure out in the same way as in the launch.
2. Brainstorm other ways the pot might work. Create a list for students to refer to.
3. Ask students to invent their own rules for the magic pot. Students could be in groups of two or work individually. Make a T-chart with 8 pairs of numbers on one side of a sheet of paper and the rule on the other side using an equation.

Summarize: Students will share their t-charts and the class should suggest equations that describe the pattern. Some patterns will generate more than one equation that could work. These are introduced as equivalent equations. Compare and contrast patterns. Reinforce use of the same symbol for a value that is the same. Vocabulary to reinforce: variable, equivalent equations, equations, t-chart, output, input.

Modifications: Suggest students use a one step rule. Allow students to work with partners to create the rules. Allow the use of calculators if needed.
Extensions: Suggest students use fractions, decimals or negative numbers in their patterns. Suggest students create patterns using division or two or more steps.

Checking for Understanding (Formative Assessment)

- **What will you assess?**
  - Understanding of the use of variables to describe patterns
- **How will you assess it?**
  - Use a t-chart of a function with 4 values on it and ask students to add 4 more values, describe the pattern in words, and write the equation as a homework prompt.

Reflection After Teaching the Lesson

- How did the students perform?
- How will you use this information to guide future instructional decisions?

TM-12

MDP for Functions

Distributed Practice and Questions: Grade Level ____

Big Idea(s)

Algebraic thinking is useful to describe functions in words, symbols and graphs.

MDP Activity 1	MDP Activity 2	MDP Activity 3	MDP Activity 4	MDP Activity 5
IN | OUT 3 | 6 4 | 8 5 | 10	IN | OUT 5 | 26 6 | 31 7 | 36	IN | OUT 25 | 25 26 | 26 5190 | 5190 ½ | ½	IN | OUT 4 | 7 5 | 7 6 | 7 1 | 7	IN | OUT ½ | 0 0.56 | 0.06 ¾ | ¼ 0 | -1/2
Questions: What expression could describe this function? Is there another one? How are they related?	Questions: What expression could describe this function? Is there another one? How are they related?	Questions: What expression could describe this function? Is there another one? How are they related?	Questions: What expression could describe this function? Is there another one? How are they related?	Questions: What expression could describe this function? Is there another one? How are they related?

Activity 9 – Processing Readings

Time: 30 minutes

Activity Synopsis:
Instruct participants to create a metaphor for connections and design a quick picture of it.

Introduction to the Activity:
Complete this sentence:
Helping children make connections in mathematics is like (insert an activity or something people do) because
_____. Post on wall for a gallery walk later.

Materials:
Poster pads
PSSM reading on Connections
Article on equality
TM-4 Reflection questions for Year 2 Day 1 (overhead)

TM 4 Reflection Questions from Day 1 Reading Assignment

1. How well is your current school practice addressing these principles? How important is equity?

2. After reading the many modifications available, what are three strategies you would recommend to classroom teachers to meet students' needs? Explain your reasons for your choices.

3. What are the implications for students who do not grasp the fundamental concept of equality?

Activity 10 – Reflection / Homework Assignments

Time: 15 minutes

Introduction to the Activity:
Describe homework and reading assignments for next time for participants.

Materials:
TM-13 Reading Assignment for Day 2
TM-14 Practice Assignment for Day 2

TM 13 Reading Assignment for Day 2

1. The principle of focus for Day 2 is Technology.
   - **Read** the section in the PSSM on the Technology Principle for Grades K-12 in *Principles and Standards for School Mathematics*, pages 24-27.
   - **Reflection Question**: How well is your current school practice addressing this principle? How important is using technology?
2. Using assessment in new ways requires new skills. Read about how teachers have begun doing this.
   - **Read** Chapter 7 "Listening to Children: Informing Us and Guiding Our Instruction" in *Teaching Mathematics through Problem Solving Grades PreK-6*, pages 107-121.
   - **Reflection Question:** What do you think is meant by "we (teachers) hear what we understand not understand what we hear"? How can we become better listeners to our children?
3. Geometry will be the content standard focus for Days 2-4. We will read it in sections.
   - **Read** "Geometry Standard" for Grades Pre-K – 12 (pp. 40-42) and first subskill "Analyzing characteristics of two-and three-dimensional geometric shapes …" for Grades K-2 (p. 96-97) and Grades 3-5 (pp. 164-166) in *Principles and Standards for School Mathematics*.
   - Be prepared to analyze the Day 3 activities in terms of the first subskill of the Geometry Standard.
   - **Reflection Question**: According to these readings, what is the teacher's role in developing reasoning and proof in geometry?

TM-14 Practice Assignment for Day 2

Examine your instructional tasks for one week. How many are at the Doing Mathematics level or Procedures with Connections? If none are, find a task you could try with students from the provided packet or one of the resources we looked at on the Integrating Standards Website. If you are using a problem-based text, use one or more of the lessons in your program.

Do the following:

1. Choose a task you judge at either the doing mathematics or procedures with connections level. Explain why you think it is at that level.

2. Teach the lesson and collect samples or observation notes of student work that reflect the range of ability in the classroom.

3. Try to determine the level of understanding the sample students have of the topic. What new insights can you gain from the students' work or discussion that this approach allowed to surface?